Seven Frieze Designs

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**Frieze Pattern 1**

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**Frieze Pattern 2**

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**Frieze Pattern 3**

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**Frieze Pattern 4**

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**Frieze Pattern 5**

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**Frieze Pattern 6**

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**Frieze Pattern 7**

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The seven frieze groups

**p111:** This has only the translation symmetry.

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**p1m1:** This has a horizontal mirror symmetry.

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**pm11:** This has vertical mirror symmetries spaced half the translation length.

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**pmm2:** This has vertical mirror symmetry, horizontal mirror symmetry, and 180° rotations where the mirrors intersect.

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**p112:** This has only 180° rotations, spaced at half the translation distance.

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**p11a:** This has a glide reflection, half the length of the translation.

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**pna:** This has a glide reflection, with alternating vertical mirrors and 180° rotations.
# The Seven Frieze Groups

<table>
<thead>
<tr>
<th>Name</th>
<th>Line Example</th>
<th>Symmetry elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hop</td>
<td></td>
<td>Only translation</td>
</tr>
<tr>
<td>Jump</td>
<td></td>
<td>Flip over horizontal</td>
</tr>
<tr>
<td>Walk</td>
<td></td>
<td>Glide reflection</td>
</tr>
<tr>
<td>Sisle</td>
<td></td>
<td>Two flip over verticals</td>
</tr>
<tr>
<td>SpinHop</td>
<td></td>
<td>Two rotate 180s</td>
</tr>
<tr>
<td>SpinSisle</td>
<td></td>
<td>Sisle + SpinHop</td>
</tr>
<tr>
<td>SpinJump</td>
<td></td>
<td>Everything</td>
</tr>
</tbody>
</table>

Draw the symmetry elements: lines of reflection and centers of 180 degree rotations (half-turns).

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## Frieze Patterns

### p111
The only transformation in this pattern is a

![Image](https://www.msu.edu/user/kaplan14/Publish/page21.html)

### p1m1
The transformation in this pattern is a reflection across a horizontal mirror.

![Image](https://www.msu.edu/user/kaplan14/Publish/page21.html)

### pmm2
The transformations in this pattern are three reflections, one across a horizontal mirror and the other two across parallel vertical mirrors and two half-turns.

![Image](https://www.msu.edu/user/kaplan14/Publish/page21.html)

### p1a1
The transformation in this pattern is a glide reflection.

![Image](https://www.msu.edu/user/kaplan14/Publish/page21.html)

### pma2
The transformations in this pattern are a reflection across a vertical mirror and a half turn.

![Image](https://www.msu.edu/user/kaplan14/Publish/page21.html)

### p112
The transformations in this pattern are two half turns.

![Image](https://www.msu.edu/user/kaplan14/Publish/page21.html)

### pm11
The transformations in this pattern are two parallel reflections across vertical mirrors.

![Image](https://www.msu.edu/user/kaplan14/Publish/page21.html)
Short crystallographic notation for frieze patterns

The seven types of periodic frieze patterns have been given various labels; one frequently-used system is based on notation used by crystallographers. Each label consists of two symbols, and is assigned to a periodic frieze pattern according to the following rules:

- The first symbol is * if the pattern has vertical reflection symmetry, otherwise the first symbol is 1.
- The second symbol is * if the pattern has midline reflection (horizontal reflection) symmetry.
- If the pattern has no midline reflection symmetry, but does have glide-reflection symmetry, the second symbol is g.
- If the pattern has no midline reflection symmetry or glide-reflection symmetry, but does have half-turn symmetry, the second symbol is 2.
- If the pattern has no midline reflection symmetry, no glide-reflection symmetry, and no half-turn symmetry, the second symbol is 1.

Thus the seven labels to identify the symmetry type of a periodic frieze pattern are 11, ml, 12, lg, lm, mg, and mm.

Exercise: Use the rules above to develop a series of questions that have yes-no answers so that by asking the questions in order, you can assign one of the labels above to a periodic frieze pattern. Then find the crystallographic label for each of the patterns below. (more on back)

In addition to the crystallographic notation, there is orbifold notation for frieze patterns. Also, John Conway has names that describe how to produce each type of pattern with footprints; his notation is next to seven of the patterns below (the quadrilateral represents a foot). Assign the correct crystallographic and orbifold notation to each of the periodic frieze patterns below.

<table>
<thead>
<tr>
<th>Crystallographic</th>
<th><em>2</em>2</th>
<th>2*2</th>
<th><em>2</em>2</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Orbifold</td>
<td><em>2</em>2</td>
<td><em>2</em>2</td>
<td><em>2</em>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

hop

dally

dizzy

dizzy

dizzy

dizzy

dizzy

§ 8. Conway (a mathematician at Princeton) devised the following names for the different frieze groups. Determine the rigid motions of each of the 7 patterns below:

(a) Hop

(b) SpinHop

(c) Jump

(d) Side

(e) Step

(f) SpinJump

(g) SpinSide

Possibilities for border patterns are endless. Surprisingly, based on symmetry, any border naturally falls into one of seven possible types.

Before going into the classification scheme, a variety of borders are given below. First illustrations of each of the seven types are given followed by selections of representative African and Native American borders.

Seven Border Types
Border Patterns: The Seven Types

1m
Centerline Reflectional Symmetry.

1g
Glide Reflectional Symmetry

12
Half-turn Symmetry.

11
Translational Symmetry only.

mm
Centerline and Crossline Reflectional Symmetry.

mg
Crossline and Glide Reflectional Symmetry.

m1
Crossline Reflectional Symmetry
Borders

Eye-catching linear designs framing doorways or adorning buildings enrich architecture. Symmetrical swirls or the tease of near symmetry make borders a favorite device to entice our attention in magazines, newspapers and consumer packaging. Border motifs can evoke social or psychic overtones through their ethnic or personal associations. Attractive designs might occur spontaneously when our idle doodles are repeated as in a border.

Decorative borders are everywhere, an expression of the pleasure we find when surrounded by the beauty of symmetry.

We will use the term border pattern for a design which can naturally be continued indefinitely in both directions along a line. By their nature border patterns exhibit translational symmetry. Border patterns are also sometimes called frieze patterns.

Different cultural traditions are suggested by these examples.

African Border Designs

Sources:

- Geoffrey Williams, African Designs from Traditional Sources, Dover.
- David Crow, Symmetry, Rigid Motions, and Patterns, COMAP
Southwestern Native American Designs

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 
10. 
11. 
12. 

Source: Dorothy Smith Sides, Decorative Art of the Southwestern Indians, Dover.