

Title:

Study Guide Theme: Pollution

Featured Photos

Water Well



(India) Water Well: Crowding around a communal well in India, people gather to collect water from a huge well in the village of Natwarghad in the western Indian state of Gujarat on June 1, 2003. Natwargadh is in the midst of the worst drought in over a decade. Dams, well, and ponds have gone dry across the western and northern parts of Gujarat, forcing people to wait for hours around village ponds for the irregular state-run water tankers to show up as the temperature soars to over 111 degrees Fahrenheit. [credit: Amit Dave]

Train Station



(Bangladesh) Train Station: Mass crowds are commonplace ... Bangladeshi passengers sit on the roof and sides of an overcrowded train as others wait on the roof of the platform as they try to reach their homes to celebrate Eid al-Adha at the Airport train station, outskirts of Dhaka, Bangladesh, Nov. 16, 2010. [credit: Pavel Rahman]

Sao Paulo, Brazil



(Brazil) Sao Paulo, Brazil: The Paraisopolis favela borders the affluent district of Morumbi. In 2004, photographer Tuca Vieira captured the image of the Paraisopolis favela next to its wealthy neighbor, Morumbi, that came to symbolize the

gap between Sao Paulo's rich and poor. [credit: Tuca Vieira]

Overview: Students will examine the concept of density through photos dealing with population density and its effects. Students will first discuss density through a conceptual lens with which they may be familiar – namely, science/chemistry. Given how far this discussion can go – namely, if students know that $density = (mass)(volume)$ or if we need to create from scratch a conceptual understanding – will inform how we need to approach the discussion of population density (making ‘people’ replace ‘mass’, as well as ‘land area’ replace ‘volume’, though being careful to ensure that students understand the difference between ‘area’ and ‘volume’ as a concept; that we are not making them synonymous here, only using one to replace the other in this context). As such, there must be some flexibility in the lesson, how it starts and how it progresses; it will be necessary to plan for both options. However, whatever the prior knowledge, students will develop an understanding of population density, and from there, discuss environmental impacts and correlations – if there are any – between population density and pollution.

Grade level(s): 10th, 11th

Subject(s): Math III/Integrated III/Algebra II

Corresponding National Standards (from [National Council of Teachers of Mathematics](#) Process & Content Standards):
Content Standards

Algebra:

Understand Patterns, Relations, and Functions

- In grades 9-12, each and every student should interpret representations of function of two variables

Represent and Analyze Mathematical Situations and Structure Using Algebraic Symbols

- In grades 9-12, each and every student should understand the meaning of equivalent forms of expressions, equations, inequalities and relations
- In grades 9-12, each and every student should use symbolic algebra to represent and explain mathematical relationships
- In grades 9-12, each and every student should judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology

Use Mathematical Models to Represent and Understand Quantitative Relationships

- In grades 9-12, each and every student should draw reasonable conclusions about a situation being modeled

Data Analysis & Probability

Formulate Questions that can be Addressed with Data and Collect, Organize, and Display Relevant Data to Answer Them

- In grades 9-12, each and every student should understand the differences among various kinds of studies and which types of inferences can legitimately be drawn from each
- In grades 9-12, each and every student should understand histograms, parallel box plots, and scatter plots and use them to display data

Process Standards

Problem Solving

- Instructional programs from prekindergarten through grade 12 should enable each and every student to –
 - o Build new mathematical knowledge through problem solving
 - o Solve problems that arise in mathematics and other contexts

Reasoning and Proof

- Instructional programs from prekindergarten through grade 12 should enable each and every student to –
 - o Make and investigate mathematical conjectures

Communication

- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others

- Analyze and evaluate the mathematical thinking and strategies of others

Connections

- Instructional programs from prekindergarten through grade 12 should enable each and every student to –
 - o Recognize and apply mathematics in contexts outside of mathematics

Representation

- Instructional programs from prekindergarten through grade 12 should enable each and every student to –
 - o Use representations to model and interpret physical, social, and mathematical phenomena

Corresponding Global Competency Skills: (<https://asiasociety.org/education/what-global-competence> - from Global Competence Matrix for Mathematics)

- Investigate the World
 - o Students can identify an issue or question of global or local significance that calls for a mathematical approach
 - o Students can select an appropriate mathematical model or approach that fits a given question
 - o Students can interpret and apply the results of mathematical analysis to defend and argument about a globally significant problem
- Recognize Perspectives
 - o Students can recognize and express how mathematics helps develop their understanding of the world
- Communicate Ideas
 - o Students can communicate mathematical thinking coherently to diverse audiences of peers, teachers, and members of the community
 - o Students can use the language of mathematics, mathematical representations, and statistics to organize, record, and communicate mathematical ideas precisely
- Take Action
 - o Identify how mathematics can be used to understand or explain a local or global issue
 - o Use connections among mathematical ideas to take action on local and global issues
 - o Reflect how mathematics contributes to their capacity to advocate for improvement

Essential Question(s)

- What is density?
- How does density apply conceptually to human beings?
- Does population density influence environmental concerns? If so, which ones? Support your answer.

Specific Strategies and Activities by Grade Level:

Open Class – What is density?

- Assess any background knowledge students have
 - o Includes science understanding (density = $\frac{mass}{volume}$ [shorthand $D = \frac{M}{V}$])
 - Deal with any other interpretations students may know ($M = VD$ or $V = \frac{M}{D}$) and how (if) they are synonymous expressions of the relationships between these variables
 - o If no formal science understanding, students may use less formal language to describe (i.e. ‘how much can be crammed into something’)
 - o Can use visuals to demonstrate this
 - Either way, have students stand up in groups/clusters and ask students to assess density in that situation
 - o Establish definition,, if not already known ($D = \frac{M}{V}$), and practice problems:
 - Ex. If a cube with side length 4 cm has a mass of 160 g, find the cube’s density.

- Must find volume: $V = 4 \times 4 \times 4 = 4^3 = 64 \text{ cm}^3$
 - $D = \frac{M}{V} = \frac{160 \text{ g}}{64 \text{ cm}^3} = 2.5 \frac{\text{g}}{\text{cm}^3}$
 - Ex. An object with density $20 \frac{\text{kg}}{\text{m}^3}$ has a mass of 45 kg. What is the volume of the object?
 - $V = \frac{M}{D} = \frac{45 \text{ kg}}{20 \text{ kg/m}^3} = 2.25 \frac{\text{kg} \cdot \text{m}^3}{\text{kg}} = 2.25 \text{ m}^3$
 - Ex. Object A has a volume of 75 cm^3 , while Object B has a volume that is $\frac{2}{3}$ that of Object A. If Object A has a mass of .07 kg and Object B has a mass of 60 g, which object is less dense?
 - Ex. Convert units of mass all to grams: Object A mass = 70 g; Object B volume = $\frac{2}{3}(75 \text{ cm}^3) = 50 \text{ cm}^3$
 - Object A: $D = \frac{M}{V} = \frac{70 \text{ g}}{75 \text{ cm}^3} = \frac{14}{15} \frac{\text{g}}{\text{cm}^3}$ or $.933 \frac{\text{g}}{\text{cm}^3}$ ← Less Dense
 - Object B: $D = \frac{M}{V} = \frac{60 \text{ g}}{50 \text{ cm}^3} = \frac{6}{5} \frac{\text{g}}{\text{cm}^3}$ or $1.2 \frac{\text{g}}{\text{cm}^3}$
 - Have students stand in groups to demonstrate the density differences between Object A and Object B (stand further apart in group A than group B to show less dense) ← demonstrates they understand density conceptually
 - Let this transition into photos to discuss population (people) density
- **Water Well**
- Put students in groups (3-4), have students retrieve ChromeBooks from cart (1 ChromeBook per student is ideal, but a group can share, depending on resources available)
 - Ask: Can you determine the density in this situation? (~5-10 minutes)
 - Students may ask, “Density of what?” – Answer: the population of the picture
 - What is the population? Where is this?
 - Have students discuss where they think this photo is and what they think is happening here
 - Give background information of the picture
 - Communal well in India (locate India on Google maps for students to orient themselves)
 - Introduce *Population Density* – the number of people living in a unit of area (~25-35 minutes)
 - All of these people are living in this area
 - Estimate the population density of this picture
 - As a group, students will estimate the area covered
 - Some will use a square/rectangle
 - Some will realize that the area of the open well isn’t ‘populated’, therefore that area should not figure into the calculation (are of square/rectangle – area of circle)
 - Students will estimate the number of people present
 - How do students translate this area and population estimate into a density?
 - Start with $D = \frac{M}{V}$, what does ‘mass’ represent and what does ‘volume’ represent?
 - Let students discuss in groups and come to a conclusion (5 min)
 - Let groups debate their conclusions (5 min)
 - Reach a class consensus (5-15 min)
 - Mass = population
 - Volume = area
 - Be VERY careful here – students must understand that area and volume are not equivalent concepts

- We need ‘area’ in place of ‘volume’ because people, standing on an 2-dimensional *area* take up 3-dimensional space, which is *volume*
 - If students struggle to understand this, have students stand and cluster up within a given space – the space upon which they stand is the area, but they – as humans – are taking up 3-D space (volume)
 - Depending on how much students struggle with relating density ($\frac{M}{V}$) to $\frac{\text{population}}{\text{area}}$, this portion of the lesson may take up to 15 minutes – prep accordingly
 - Have each group calculate the density based on their estimates, then have each group report out
 - As a class, determine which estimate the class deems most accurate and why
 - Compare their calculation with the population density given on data.Worldbank.org for 2017: 450.419 people/sq km of land
 - Compare their answers (given in standard units) to the World Bank answer (given in metric) → convert
- **Train Station**
 - Train station in Dhaka, Bangladesh (show on Google maps so students can orient themselves)
 - Have groups calculate an estimate for population density in the train station
 - Consider questions about people sitting on top of train – do they figure into the calculation?
 - Yes
 - Do we add the area of the train roof to the area of the ground?
 - No – it’s all considered one area, even if people are stacked on top of each other
 - Compare group estimates with data.Worldbank.org for 2017 in Bangladesh: 1,265.036 people/sq km
 - Convert to assess how close our approximation was
- **Sao Paulo, Brazil**
 - Based on the understanding that adding population vertically in a given area doesn’t increase the area (even if there is man-made ‘floor’ space upon which those people are placed), determine which space has the greater population density: Paraisopolis Favela (left half) or Morumbi District (right half)? Why?
 - Let groups discuss/debate amongst themselves (~5 min), then have groups discuss/debate with each other
 - We honestly don’t know the answer, but considering that the apartment building is stacking up (vertically) on a given piece of land (area), it is likely that the apartment building has the greater population density
 - Caveat: How many live in one apartment compared to how many live in one home in the favela
- American equivalents: apartment buildings in cities and college dormitories
 - NYC → The Bronx



Credit for Visuals: *These images were created by the study guide author, Stephanie Morgan, in July 2018. As property of the author, the photo is authorized to be used as part of this study guide/lesson.*

*This study guide was created by Stephanie Morgan, Pisgah High School, as part of the 2018 World View Fellows Program: The OVERBook Project on the Environment and Sustainability
For more information about the program, please visit <http://worldview.unc.edu/>*

- Look at apartment buildings in foreground and background – discuss occupancy, size of a city block, population density in this area
- College dormitory: UNC Chapel Hill → Carmichael Hall (dormitory) → approximately 84 rooms per floor, 5 floors, assume double occupancy = 840 ppl



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- Does population density influence environmental concerns? If so, which concerns? Support your answer.
 - o Take student answers – they may discuss/debate the different issues as issues are suggested
 - Hopefully air pollution will arise – if not, we'll look at it directly now
 - o Look at abstract from [Air pollution and population health: a global challenge](#)
 - Is 'population density' every mentioned? Is it implied?
 - Urbanization and rapid population growth could contribute to population density
 - So is 'population density' the cause?
- Class discussion: Teacher goes to GapMinder.com (projected for the students to follow along – students should follow along on individual Chromebooks, or with a partner/group depending on number of laptops available to be used) → select 'Tools'
 - o If the 'bubble graph' (looks like a scatter plot with different sized dots/bubbles) doesn't show up first, use [https://www.gapminder.org/tools/#\\$chart-type=bubbles](https://www.gapminder.org/tools/#$chart-type=bubbles)
 - o On the x-axis, be sure 'linear' is selected, then click the label, search 'population density', and let that be the label on the x-axis
 - o On the y-axis, be sure 'linear' is selected, then click the label, click 'CO2 emissions'
 - o This will produce a graph that is clustered near the origin
 - With the tools on the right side of the page, click the magnifier icon that will zoom in, then click and drag a box around the cluster of dots – this will change the graph scale and zoom in on those points
 - Repeat until the graph has zoomed in as much as you desire
 - Press 'play' and what the information change over time
 - o Can locate India, Bangladesh, Brazil, and/or the United States (click the countries indicated on the right hand side of the webpage) to compare those
 - Based on what we know of graphs and models, does there appear to be relationship between population density and CO2 emissions?
 - Not really → the US has the largest CO2 emissions amounts of the four, but barely has a larger population density than Brazil, definitely smaller than India and Bangladesh
 - Does that mean there is no correlation?
 - o Not necessarily – could have *confounding variables*, things that influence the relationship that are related to population density and pollution
 - o This is alluded at in the abstract we read
 - Students: There are lots of options available for use on the x-axis – select different categories for the x-axis and determine if a relationship with CO2 emissions is more readily visible
 - Write down at least 5 labels that appear to produce a visible mathematical relationship
 - Describe that relationship (linear, exponential, etc)
 - o Possible options include:
 - life expectancy (linear or exponential growth, depending on how they zoom)
 - income (linear growth)
 - babies per woman (exponential decay)
 - extreme poverty (exponential or linear decay, depending on how they zoom)
 - GDP/employee (linear growth)
 - GNI/per capita (exponential growth)
 - human development index (exponential growth)
 - at least basic water source/overall (may be argued as exponential)

- growth)
 - at least basic sanitation/overall (may be argued as exponential growth)
 - fixed line subscribers (linear growth)
 - individuals using the internet (linear growth)
 - sulfur emissions per person (exponential growth)
 - energy use per person (linear growth)
 - oil consumption per person (linear growth)
 - electricity use per person (linear growth)
 - residential electricity use per person (linear growth)
 - As a group, pick 1-2 relationships that you find most interesting and be prepared to present it to the class (type of relationship, what it tells us about the situation, etc)
 - Cannot present a relationship that another group has discussed
 - Teacher will pass out conclusions to two articles:
 - The Impacts of Population Density, and State & National Litter Prevention Programs on Marine Debris; Brogle, M
 - Effects of Affluence and Population Density on Waste Generation and Disposal of Municipal Solid Wastes; Matsunaga, K. & Themelis, N.
 - Students will read these on their own, then start assignment (the readings will tie into a part of their assignment for the day)
- Summary:
- Assignment:
 - Density Word Problems, 1-13 (adapted from Mr. Trent Thompson's [Density Word Problems](#) assignment, Chippewa Hills Intermediate School, www.chsd.us/~tthompson/Science%208/Assignments/Tri%201/Density%20Word%20Problems.pdf – attached below)
 - Assessing what your group discovered as well as what other groups presented and the articles you read, answer the following question (1-2 paragraphs, complete sentences): *What overarching factor(s) that we have seen today do you believe is/are more directly correlated to pollution than population density (or that go hand-in-hand with population density)? Explain.*

Materials:

ChromeBooks/Laptops

Teacher computer that can connect to internet and projector

Density Word Problems worksheet

OVERBook Citations:

Dave, A. (2003, June 1). *Water Well* [Photograph]. In Tom Butler (Ed.), *Overpopulation, Overdevelopment, Overshoot* (p. 47). San Francisco, CA: The Foundation for Deep Ecology. *This photograph is used in accordance with the guidelines of the 2018 World View Fellows Program: The OVERBook Project on the Environment and Sustainability.*

Rahman, P. (2010, Nov 16). *Train Station* [Photograph]. In Tom Butler (Ed.), *Overpopulation, Overdevelopment, Overshoot* (pp. 114-115). San Francisco, CA: The Foundation for Deep Ecology. *This photograph is used in accordance with the guidelines of the 2018 World View Fellows Program: The OVERBook Project on the Environment and Sustainability.*

Vieira, T. (2004). *Sao Paulo, Brazil* [Photograph]. In Tom Butler (Ed.), *Overpopulation, Overdevelopment, Overshoot* (p. 52). San Francisco, CA: The Foundation for Deep Ecology. *This photograph is used in accordance with the guidelines of the 2018 World View Fellows Program: The OVERBook Project on the Environment and Sustainability.*

Articles & Links:

Brogle, M. (2012). The impacts of population density, and state & national litter prevention programs on marine debris. *University of South Florida Scholar Commons: Graduate Theses and Dissertations*, University of South Florida. Retrieved from <http://scholarcommons.usf.edu/cgi/viewcontent.cgi?article=5189&context=etd>.

[The impacts of population density, and state & national litter prevention programs on marine debris](#)

Chen, B. & Haidon, K. (2008). Air pollution and population health: a global challenge. *Environmental Health & Preventative Medicine*, 13(2), 94-101. Retrieved from <https://doi.org/10.1007/s12199-007-0018-5>.

[Air pollution and population health: a global challenge](#)

Matsunaga, K. & Themelis, N. (n.d.). Effects of affluence and population density on waste generation and disposal of municipal solid wastes. *School of Engineering and Applied Science*, Columbia University. Retrieved from www.seas.columbia.edu/earth/waste-affluence-paper.pdf.

[Effects of affluence and population density on waste generation and disposal of municipal and solids wastes](#)