Title:

Study Guide Theme: Water

Featured Photos

Aftermath

(Japan) Aftermath: A 2011 tsunami prompted a nuclear meltdown at the Fukushima Daiichi Nuclear Station in Japan, galvanizing the world's attention. Lesser known was tsunami-related damage to Japan's fossil fuel energy infrastructure, including this facility near Tokyo. [credit: AFLO]

Trash Wave

(Indonesia) Trash Wave: Indonesia surfer … Surfing trash island: photographer captures startling images of garbage-strewn waves in Indonesia. Indonesian surf champion Dede Suryana rides a wave filled with trash on Untung Jawa Island. 2013. [credit: Zak Noyle]

Man Bathing

(India) Man Bathing: A larger percentage of the global population … A man bathes from a broken water pipe line in Noida slum, located in the northern Indian state of Uttar Pradesh June 10, 2011. [credit: Parivartan Sharma]

Overview: Students will address water use through the analysis of photos. In the process, students will be given background on the photos and the situations represented, will assess the water impacts in each scenario, and students will use Calculus methods to analyze things that are occurring in the photos through modeling of position, velocity, and acceleration (ideas which will be introduced in the lesson to build on applications of derivatives). Students will analyze position, velocity, and acceleration graphs for each model and note, in context, what is happening in each graph and how it relates back to the scenario photographed. The third photo (man bathing) will specifically be linked back to more local water related issues and concerns (Central Valley, CA; Flint, MI; Duke Power Coal Ash Spill, Eastern NC; Kingston Fossil Plant, TN).

Grade level(s): 11th, 12th

Subject(s): AP Calculus

Corresponding National Standards (from College Board Mathematical Practices for AP Calculus (MPACs)):
MPACs

- MPAC 2: Connecting Concepts
    - b) Students can use the connection between concepts or processes to solve problems
    - c) Students can connect concepts to their visual representations with or without technology
- MPAC 3: Implementing Algebraic/Computational Processes
    - a) Students can select appropriate mathematical strategies
    - b) Students can sequence algebraic/computational procedures logically
    - c) Students can complete algebraic/computational processes correctly
    - f) Students can connect the results of algebraic/computation processes to the question asked
- MPAC 4: Connecting Multiple Representations
    - c) Students can identify how mathematical characteristics of functions are related in different representations
- MPAC 5: Building Notational Fluency
    - d) Students can assign meaning to notation, accurately interpreting notation in a given problem and across different contexts
- MPAC 6: Communicating
    - c) Students can explain the meaning of expressions, notation, and results in terms of a context (including units)
    - d) Students can explain the connections among concepts

Corresponding Global Competency Skills: (https://asiasociety.org/education/what-global-competence - from Global Competence Matrix for Mathematics)

- Investigate the World – Students investigate the world beyond their immediate environment
    - Identify issue and frame researchable questions of local, regional, or global significance that call for or emerge from a mathematical or statistical approach
    - Select or construct appropriate mathematical or statistical models or approaches to address globally significant researchable questions
- Recognize Perspectives – Students recognize their own and others' perspectives
    - Recognize and express their own perspective and understanding of the word, and determine how mathematics and statistics influence and enhance that perspective and understanding
- Communicate Ideas – Students communicate their ideas effectively with diverse audiences
    - Select and use appropriate technology and media to model, analyze, represent, and communicate

    mathematical ideas for diverse audiences and purposes

- Take Action – Students translate their ideas and findings into appropriate actions to improve conditions
    - Reflect on how mathematics and statistics contribute to their capacity to advocate for local, regional, and/or global improvement

Essential Question(s)

- In what ways do we use water? (Understanding that 'we' is representing themselves as well as a collective pronoun for society at large)
- Are there ways in which our water use can be hindered? (Asked after the presentation of each photo, either as a question to students – such as what happens to the water used to put out the fire at the Fukushima plant in Aftermath & in response to students' initial reactions to Trash Wave and Man Bathing)

Specific Strategies and Activities by Grade Level:

Day 1

- Independent answer to the following question: In what ways do we use water? (~3 min)
    - Again, this is with the understanding that 'we' can include anyone, so this means individually, societally, vocationally, etc
    - Students share responses with the class, one volunteer to come keep a list on the whiteboard
        - Anticipated potential responses: drinking, cooking, recreation, transportation, electricity, hydraulics, protection (either as a defensive measure (moat) or for fighting fires), etc
- Begin photo analysis by asking students to break into small groups (3-4 students)
    - Start with **Aftermath**
    - Within groups, have students describe what is happening in the picture and tie it back to something on their list of water uses (if that use is not on the list, teacher will add it) (~5 min)
        - Is this a water use with which we are familiar? Why or why not?
            - This should lead into a brief discussion of similarities (firefighting) and potential differences (we are not near a large body of water and therefore do not frequently see fire*boats* in action)
    - Teacher will give students background of the situation (~5 min)
        - Pull up Google Maps and pin Fukushima, Japan, in order to orient students to its location
        - Fukushima, Japan – after the tsunami that prompted a nuclear meltdown at the Daiichi Nuclear Station, there was also lesser-known damage done to the fossil fuel energy structure of Japan, including at this facility also in Fukushima
    - Modeling (<35 min)
        - In groups, have students use iPads/Chromebooks to look up dimensions and specifications of fireboats
        - Have students focus on the furthest left fireboat in the photo, as well as the stream of water coming from its lowest canon
            - Estimate how far water coming from the bottom canon will travel
        - Have groups report out their findings and estimations
            - Have classroom discussion about which data we want to use to best estimate the scenario
            - Once a consensus is reached on estimates, create a table of values upon which regression can be performed
            - Students will individually perform the regression using a calculator – groups will decide regression model (linear, quadratic, cubic, etc) to use and must be able to justify its use
                - If using TI-83 or TI-84 models, remind students to turn on StatDiagnostics; not necessary for TI-Nspire

- Students will report out findings (models and r^2 values)
  - Class will analyze which model is best using r^2 values
- Instruction (students can remain in groups)
  - The model created is the **position** function
    - Typically labeled *s(t)*, occasionally seen as *x(t)*
    - This is position (y) against time (x)
    - Using graphing technology, students will graph the function and explain what is happening to a water droplet (based on intuition, the photo, and the graph) at any given time, *t*
  - Introduce **velocity** (~25 min)
    - Velocity = first derivative of the position
      - What is velocity?
        - As a first derivative, it is the rate of change of the original function, so it is the rate of change of the position
          - Velocity units: $\frac{distance\ units}{time\ units}$
        - Sometimes called *speed*, but this is an oversimplification, as velocity gives rate of change of position <u>and</u> indicates direction based on the sign
      - Notationally, *v(t) = s'(t)*
      - Students will calculate *v(t)* by hand individually, then check their answers against those found by others in their groups before reporting out to the class
      - Recall what was learned about how first derivatives relate back to the original graph
        - Where the first derivative is positive, negative, or zero related to the original function (critical points); increasing or decreasing (concavity)
          - Can we determine a time, *t*, when the velocity is 0? What does this represent on the position, *s(t)*, function?
            - Answer: The time *t* will be based on the function the class creates, but it will be located where *v(t)* crosses the x-axis and *s(t)* is the critical point (maximum) on the quadratic
        - Now graph *v(t)*
          - What do you notice about time, *t*, where the velocity is equal to 0? Why does this happen?
            - Answer: The first derivative is 0 where the original function changes from increasing to decreasing or decreasing to increasing. Because *s(t)* changes from increasing to decreasing (max) at that time, *v(t)* must cross the x-axis at this time, *t*. The first derivative is a linear, decreasing graph because the derivative of a quadratic with $a < 0$ must be a linear function with a negative slope (rate of change)
              - This is further backed up by concavity – the original function is always concave down, implying the first derivative will always decrease
      - Questions?
  - Introduce **acceleration** (20 min)

- Acceleration, *a(t)* = rate of change of velocity
  - But isn't velocity a rate of change for position?
    - Yes – so acceleration is the rate of change *of* a rate of change
    - Acceleration units: $\frac{\frac{distance\ units}{time\ units}}{time\ units} = \frac{distance\ units}{(time\ units)^2}$
    - Physics example – acceleration due to gravity: 9.8 m/s/s (9.8 m/s^2) or 32.2 ft/s/s (32.2 ft/s^2)
  - Ask students: How does acceleration relate back to position?
    - Answer: Second derivative of *s(x)*
    - Therefore, *a(t)* = *v'(t)* = *s''(t)*
  - Students will calculate *a(t)* by hand, then compare their answer with what group members calculated before reporting out to class
  - Without using a calculator/graphing utility, what is the graph of *a(t)* going to look like?
    - Answer: Horizontal line before the x-axis
    - Why? – Because from the time the water leaves the canon, there is nothing providing it more energy to combat gravity; the water is always being pulled downward (thus a negative number)
  - Analyze such a graph based on what we know of how the second derivative ties back to the original function and first derivative
    - Original function
      - Concavity (same explanation as above)
    - 1st derivative, *v(t)*
      - Because the first derivative is always decreasing, *a(t)* will always be negative
  - Are there ways in which our water use can be hindered?
    - What happens to this water after it is used to put out these fires?
      - Future use, ramifications, etc.
  - Questions?
- Summary: What happened to the water used to put out the fires at Fukishima?
  - Questions to consider:
    - Did it run back into the harbor?
    - Is it contaminated?

- Assignment:
  - Finney, Demana, Waits, Kennedy's *Calculus: Graphical, Numerical, Algebraic* pg. 129-130 (2-6, 12-13)

Day 2

- Get into different groups than were in on Day 1 (still 3-4 students)
- Introduce **Trash Wave** image (~5 min)
  - Have students identify which category this image would go under in the water use list created the previous day (if that use is not on the list, teacher will add it)
    - Is this a water use with which we are familiar? Why or why not?
      - Similar – familiar with the concept of surfing, using the beach for recreation – students may link recreational water use to fresh water sources as well, such as lakes and rivers
      - Differences – surfing with trash?
  - Assumption is that students will have questions about the trash
    - Teacher will open up Google Map and pin Java, Indonesia, on the map for students to orient themselves to the location of this scene
    - Background: Indonesian surfer Dede Surinaya catches a wave in a remote bay on Java, Indonesia

(world's most populated island)

- Ask students: Why is there so much trash?
  - Possible answer: World's most populous island means lots of consumption, lots of trash, means improper disposal allows it into the ocean
  - Is this an isolated issue? ← leave this question to be pondered independently while modeling with calculus

- Calculus (~15 min):
  - Model
    - In groups, discuss the path the surfer travels, estimate the polynomial that best approximates the path of the surfer on the wave, and create a table of values to model the situation
    - Each student will find the regression model and r^2 value in his or her calculator and compare with others in his group
    - Within each group, students will:
      - Graph their regression model, *s(t)*
      - Calculate and graph *v(t)* and *a(t)*
    - Class discussion: Depending on how they modeled the original *s(t)*, there are lots of options for future *v(t)* and *a(t)*.
      - If *s(t)* is linear, then *v(t)* will be a constant function with a negative value and *a(t)* will be a constant function equal to 0 – what does this mean?
        - *v(t)*: *s(t)* was a decreasing linear function, so *v(t)* should be a constant below the x-axis
        - *a(t)*: *v(t)* is constant, so the *rate of change* of the velocity must be zero ($\frac{\Delta v}{\Delta t} = \frac{0}{\Delta t} = 0$)
      - If *s(t)* is a quadratic or higher degree function, can discuss what *v(t)* and *a(t)* should look like based on prior knowledge of derivatives learned earlier in this unit
- Cycle back around to the picture - Ask: Are there ways in which our water use can be hindered? (~15 min)
  - Some may not want to visit a beach where there is so much garbage – What is in that garbage? Household waste, medical waste, etc ← all are possibilities
  - Connect back to the question about trash in Java being an isolated incident - Ask students: Does this happen in the US?
    - Show students article *Hypodermic needles wash ashore on West Newport beach*
      - Pin Newport Beach, CA, in Google Maps
    - Use this article for specific references – in text and in photo – of surfers (similarities of people's water use to highlight similarity of situation and deemphasize 'othering')
      - Commonality of surfing as a pastime, participants being forced to surf in trash and medical waste in US
    - Not just a saltwater issue
  - Ask: Are beaches the only concern?
    - No – so what else? Rivers, lakes

Credit for Visual: https://pixabay.com/en/cow-drinking-lake-cattle-animal-1578950/
This image is listed as CCO Public Domain and is free for commercial use according to terms listed on the website: https://pixabay.com/en/service/terms/#usage

- Have students describe this image and any issues they observe
- Ask: Would this influence recreational use of this river? Would you want to swim in this river?
  - Class discussion
  - Cattle in rivers can cause pollution (defecate and/or urinate in water, people swimming downriver can be exposed to disease-causing microbes) → again, brings issue 'home'
- Ask: Back to that question I asked you to ponder – is water pollution an isolated issue?
  - By this point, the answer should be a resounding 'no' – hopefully have also driven home the fact that this is not 'their' (other country's) issue, but 'our' (global, human) issue

- Introduce **Man Bathing** "Good water, good life. Poor water, poor life. No water, no life." ~Sir Peter Blake (quote from the Overbook) (~15 min)
  - Ask: What are we going to model here?
  - Assumption is that students will question what is going on in the picture
    - Open Google Map and pin Uttar Pradesh, India, to allow students to orient themselves geographically
    - Background: Photo from Uttar Pradesh, India, where a man is using a broken pipe to bathe
    - Anticipated student questions:
      - Where does the pipe originate? Is it a well or spring? Some kind of city water?
      - What kind of water access does this man have at home?
      - Why is the broken pipe surrounded by trash?
    - Some of their questions could be unanswerable – what kind of water access the man may have at home, or why there is so much trash – but we do know that this is a broken water pipe and that a large percentage of the global population has limited access to clean water, so we might assume that this man is living in such a situation
  - Model (~15 min)
    - Let the questions simmer for thought while doing the math on the arc of water coming from the pipe – compare to the arc of water coming from the fireboat
      - Students will assess that this arc is smaller, so the water travels less distance, etc
      - How will this impact the velocity and acceleration?
        - Velocity and acceleration will follow the same trend in both scenarios
          - Positive velocity initially, then negative (changes at critical point)
          - Both will have negative acceleration, as this is also a parabolic model with $a < 0$
          - Velocity will decrease (Δs will be smaller in this scenario than the fireboat scenario), which means Δv will be smaller in this scenario,

causing acceleration to decrease

- Have students model, in groups, to support these conclusions
  - Groups will make a table of values, each member will model the table in a calculator and verify with group members
  - Will then calculate *v(t)* and *a(t)* to support the conclusions above

- Come back to the picture discussion, specifically the origin of the pipe – is it a spring/well or a municipal pipe? Which option is safer? (~25 min)
  - We know from background on the photo that this is a municipal pipe that has broken
  - Background information from CDC Website on Private Ground Water Wells (link below)
  - Discussion of the merits of both, specifically about assumptions of well/spring water
  - Issues with water, ground water and municipal water, recently in US:
    - Central Valley, CA – Sinking land, poisoned water: the dark side of California's mega farms
    - Flint, MI – How Flint's water crisis happened, and why it isn't over
    - Eastern NC Coal Ash Spill – The saga of North Carolina's contaminated water; EPA assists North Carolina in preventing water pollution
    - Kingston Fossil Plant, TN – 5 years after coal ash spill, little has changed; Small group turns out at TDEC to air concerns about TVA discharge permit
      - Use these articles (printed or on Chromebook/iPad), or excerpts for the longer pieces, to illustrate water (river, groundwater, and municipal water) contamination in US
      - Each group will read about one location, then report out to classmates about what is happening in the affected areas and what kind of contamination they are battling (groundwater, municipal water, etc)
      - Each group will give a brief report and take any questions

- Assignment:
  - Larson, Hostetler, Edwards' *Calculus of a Single Variable, 7th ed.* pg. 115 (91-94)
- Learning Extensions:
  - Suppose my grandmother were curious about the safety of her water. Find two local (within 100 miles of our location) companies that will test her water quality. Explain, using a minimum of one paragraph, why ensuring access to clean, fresh water is important.

Materials:

Classroom equipment and resources:

- Calculators (TI-83/84 or TI-Nspire)
- Class set of iPads/Chromebooks
- Teacher computer with internet access and projector connection
- Post-it Self Stick Easel Pad (for student presentations)

<u>News articles & links:</u>

Bharath, D. & Connelly, L. (2016). Hypodermic needles wash ashore on West Newport beach. *The Orange County Register*. Retrieved from https://www.ocregister.com
Hypodermic needles wash ashore on West Newport beach

Centers for Diseases Control and Prevention. (2014). Drinking water: Private ground water wells. *US Department of Health and Human Services*. Retrieved from https://www.cdc.gov/healthywater/drinking/private/wells/index.html

Drinking water: Private ground water wells

Crocker, B. (2017). Small group turns out at TDEC to air concerns about TVA discharge permit. *Knox News*. Retrieved from https://www.knoxnews.com
Small group turns out at TDEC to air concerns about TVA discharge permit

Gang, D. (2013). 5 years after coal-ash spill, little has changed. *USA Today*. Retrieved from https://www.usatoday.com
5 years after coal-ash spill, little has changed

Greenberg, A. (2018). Sinking land, poisoned water: the dark side of California's mega farms. *The Guardian*. Retrieved from https://www.theguardian.com.
Sinking land, poisoned water: the dark side of California's mega farms

Hobson, J. (Host) & Clark, A. (Guest). (2018). How Flint's water crisis happened, and why it isn't over [Radio program]. *Here & Now*. Boston, MA: NPR.
How Flint's Water Crisis Happened, and Why it Isn't Over

Marraccini, D. (2017). EPA assists North Carolina in preventing water pollution. *US Environmental Protection Agency.* Retrieved from https://www.epa.gov/newsreleases
EPA assists North Carolina in preventing water pollution

Semuels, A. (2017). The saga of North Carolina's contaminated water. *The Atlantic.* Retrieved from https://www.theatlantic.com
The saga of North Carolina's contaminated water

OVERBook Citations:
AFLO. (2011). *Aftermath* [Photograph]. In Tom Butler (Ed.), *Overpopulation, Overdevelopment, Overshoot* (pp. 234-235). San Francisco, CA: The Foundation for Deep Ecology. *This photograph is used in accordance with the guidelines of the 2018 World View Fellows Program: The OVERBook Project on the Environment and Sustainability.*

Noyle, Z. (2013). *Trash Wave* [Photograph]. In Tom Butler (Ed.), *Overpopulation, Overdevelopment, Overshoot* (pp. 178-179). San Francisco, CA: The Foundation for Deep Ecology. *This photograph is used in accordance with the guidelines of the 2018 World View Fellows Program: The OVERBook Project on the Environment and Sustainability.*

Sharma, P. (2011, June 10). *Man Bathing* [Photograph]. In Tom Butler (Ed.), *Overpopulation, Overdevelopment, Overshoot* (pp. 238-239). San Francisco, CA: The Foundation for Deep Ecology. *This photograph is used in accordance with the guidelines of the 2018 World View Fellows Program: The OVERBook Project on the Environment and Sustainability.*